Anomalous mapping between pionfull and pionless EFT's

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Abstract

The pion contributions to leading contact coupling of pionless EFT are studied via interactive use of non-relativistic and relativistic formulations of chiral effective field theory for nuclear forces. The dominant contribution is shown to come from a definite item in the $2N$-reducible (iteration of OPE) component of the planar box diagram, not from the $2N$-irreducible (TPE potential) component. Such anomalous mapping between pionless and pionfull EFT's occurs right within non-relativistic regime of pionfull theory. This mapping perspective may be able to shed some light on the subtle structures and renormalization of the pionfull effective field theory for nuclear forces.

1. Introduction

   Pions are the first particles known to mediate strong interactions between nucleons. After quark picture of hadrons is established, they are degraded as effective degrees–composite particles of quarks bound with gluons, and the local field theories in terms of pions and other hadronic degrees become low-energy effective field theories (EFT) of QCD with chiral symmetry being spontaneously and softly broken that are only valid in low energy processes characterized by scales well below the broken scale of chiral symmetry: $\Lambda_\chi \sim 1$ GeV. For nuclear forces, however, the direct computation using QCD is in fact impossible, where effective theories are extremely useful tools at hand. In fact, since Weinberg's seminal work in 1990[1], there have been great progresses in applying EFT methods to nucleon systems and nuclear forces in the last two decades[2–7]. In a sense, these achievements have pretty laid down the field theoretical foundation for nuclear physics. Intriguingly, there still remains an unsettled issue that is concerned with the nonperturbative treatment of pion-exchange potential[8–11]. Two prevailing choices are adopted in literature concerning this issue: (1) Nonperturbative treatment[12, 13] in numerical approach using finite cut-off a la Lepage[14] without modifying Weinberg's power counting; (2) 'Perturbative' treatment[8, 15–19] with modified power counting rules. The first choice is quite successful and efficient in phenomenology. While the second choice is also appealing due to its analytical tractability. The merits and discussions of various approaches could be found in the review articles[2–7]. The open status of this issue suggests that we are still elusive of some intricate structures of the chiral effective theory for nuclear forces. So, it is worthwhile to do further studies about the structures of the pionfull theory.

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Theoretically, it is easy to handle the pionless theory defined in much lower momentum region of nucleon scattering with pions integrated out and expanded into the contact interactions. Previously, this pionless theory has been studied without direct reference to the pionfull one as its adjacent 'underlying' theory. (In fact, the renormalization of this theory could be readily settled using a 'perturbative' scheme based on modified power counting rules[15]. Moreover, this pionless theory is also tractable entirely within nonperturbative regime thanks to the trick introduced by the Maryland group[20] with general parametrization of divergences[21–25].) Since it is an 'effective' theory of the pionfull one, it is natural to inquire what could be seen from studying the detailed mapping between the pionfull and pionless theories. Through such efforts, it is desirable to trace or be informed of the intricacy of the pion-mediated interactions and amplitudes somehow, at least some useful clues might be found from this perspective for the intricate contents and a satisfactory or efficient organization of the EFT for nuclear forces. Therefore, we start from this report on to study the issue through computing and analyzing the mapping or matching between pionfull and pionless theories for nuclear forces. We should remind that we are not attempting here at any new organization or power counting of the pionfull theory for nuclear forces, but looking at the intriguing issue from an alternative perspective that might be helpful for a satisfactory solution. Thus we will mainly work with conventional chiral effective theory in both relativistic and non-relativistic formulations[6] to see how the pionless contact interactions arise from the pion exchange diagrams. One might have anticipated that only the $2N$-irreducible diagrams be dominating such mapping into pionless EFT. As will be seen shortly, such anticipation turns out not to be true due to IR enhancement from the loop integrals appearing in the iterated diagrams or convolutions. As a matter of fact, the scales involved in low-energy NN scattering are only modestly separated, any enhancement due to convolution or loop integration may somehow twist the scale hierarchy that is required for a lucid EFT description. Thus, it is worthwhile to take closer looks at how such enhancement mechanisms affect the structures of the pionfull theory.

As a byproduct, it is also interesting to see what kind of pionless theory could be resulted from the various modified power counting schemes of pionfull theory. Thus, apart from the conventional prescription of pionfull EFT, we will also compute with the prescription recently proposed by BKV[26] to examine the consistency of the KSW power counting for pionless EFT. Basing on our closed-form solutions, we found that the empirical ERE parameters for S-channels seem to favor a scenario hosting conventional power counting[22–24], while the scenario with modified power counting seems to be dis- or less favored by the PSA data. To this end, it is interesting to see which or what scenario could be justified from the mapping of pionfull onto pionless theory. Furthermore, it is also interesting to see how the physical subset of prescription parameters [$J_{...}$] arise from the pionfull theory via matching, which is a more challenging task to be pursued in future. We feel that such perspective will also be valuable for many physical issues that are suitable for applying EFT in nonperturbative regime, especially non-relativistic EFT's with the presences of nonperturbative divergences and of IR enhancement from pinching non-relativistic poles. After all, for each EFT one could at least find one most adjacent 'underlying' theory that contains more high energy details.

This report is organized as follows. In Sec. 2, we set up the conventional pionfull and pionless Lagrangians for our use. Then in Sec. 3, we calculate the leading contact coupling induced from loop diagrams in pionfull theory. Sec. 4 will be devoted to some general discussions about our results, where the mapping using BKV prescription will also be calculated and discussed. The summary will be given in Sec. 5.

2. EFT's for *NN* scattering

2.1. Pionfull EFT

As a low-energy EFT that inherits the chiral symmetry of QCD, the pionfull theory for nuclear forces could be given in relativistic as well as non-relativistic formulation. We will work with both in an interactive manner. The relativistic Lagrangian we will use reads (following the notations of Ref.[6])

$$\mathcal{L}_{EFT(\pi)} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots, \tag{1}$$

$$\mathcal{L}_{\pi\pi} = \frac{1}{2}\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} - \frac{1}{2}m_\pi^2 \boldsymbol{\pi}^2 + O\left(\boldsymbol{\pi}^4\right), \tag{2}$$

$$\mathcal{L}_{\pi N} = \bar{\Psi}\left(i\gamma^\mu \partial_\mu - M_N - \frac{g_A}{2f_\pi}\gamma^\mu \gamma^5 \boldsymbol{\tau} \cdot \partial_\mu \boldsymbol{\pi} + O\left(\boldsymbol{\pi}^2\right)\right)\Psi, \tag{3}$$

$$\mathcal{L}_{NN} = -\left(\bar{\Psi}\Gamma_\alpha \Psi\right)\left(\bar{\Psi}\Gamma^\alpha \Psi\right), \tag{4}$$

with Γ_α being matrices constrained by Lorentz and isospin invariance. The contact Lagrangian for nucleons is as given in the pioneering work of Weinberg[1]. In non-relativistic formulation, the Lagrangian reduces to the following form using heavy baryon formalism

$$\mathcal{L}_{\pi N} = \bar{N}\left(i\partial_0 + \frac{\vec{\nabla}^2}{2M_N} - \frac{g_A}{2f_\pi}\boldsymbol{\tau} \cdot (\vec{\sigma} \cdot \vec{\nabla})\boldsymbol{\pi} + O(\boldsymbol{\pi}^2)\right)N, \tag{5}$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_0 \left(\bar{N}N\right)^2 + \cdots. \tag{6}$$

Here the contact couplings should assume the contributions from heavy mesons, etc., and scale as:

$$C_0 \sim \frac{4\pi}{M_N \Lambda_{(\pi)}}, \quad \cdots \quad (\Lambda_{(\pi)} \sim 4, 5m_\pi)$$

with $\Lambda_{(\pi)}$ being the upper scale of the pionfull EFT.

2.2. Pionless EFT

After integrating out pions and the processes above the scale of pion mass, one could further arrive at a simpler effective theory with only non-relativistic nucleon degrees and contact interactions among them:

$$\mathcal{L}_{EFT(\not\pi)} = \bar{N}\left(i\partial_0 + \frac{\nabla^2}{2M_N}\right)N - \frac{1}{2}C_0^{(\not\pi)}\left(\bar{N}N\right)^2 + \cdots, \tag{7}$$

with $\cdots$ representing other contact interactions. Now these the contact couplings in pionless theory have incorporated contributions from the pion-exchange diagrams in pionfull theory,

$$C_0^{(\not\pi)} = C_0 + \hat{T}_{NN}^{(\pi)}(\mathbf{0}, \mathbf{0}), \quad \cdots. \tag{8}$$

As pions are the lightest quanta for mediating strong forces between nucleons, it is natural to anticipate that pion-exchange diagrams should dominate the contributions to the pionless contact couplings, e.g.,

$$C_0^{(\not\pi)} \sim \frac{4\pi}{M_N \Lambda_{(\not\pi)}}, \quad \cdots \quad (\Lambda_{(\not\pi)} \sim m_\pi).$$

Below, we will study the contributions of the pion-exchange diagrams to the pionless couplings and hope that such efforts may shed some light on the intricate structures of the pionfull theory for nuclear forces.

3. Mapping into pionless EFT

In the pionfull theory, all the diagrams for NN scattering could be classified into $2N$-irreducible and $2N$-reducible ones that are traditionally defined as NN potential with pion exchanges and scattering amplitudes, respectively.

3.1. 2N-irreducible diagrams with pions: potential

The $2N$-irreducible diagrams in pionfull EFT have been computed up to next-to-next-to-next-to-leading order in literature, see Refs.[6, 13, 27, 28]. For our purpose below, it suffices to demonstrate with the one-pion exchange (OPE) and two-pion exchange (TPE) components[27]:

$$V_{1\pi}(\boldsymbol{q}) = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q}}{q^2 + m_\pi^2}, \tag{9}$$

$$V_{2\pi}(\boldsymbol{q}) = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 W_C + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 V_S + \boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} V_T, \tag{10}$$

$$\begin{aligned}
W_C &= -\frac{1}{384\pi^2 f_\pi^4}\left\{\left[4m_\pi^2\left(5g_A^4 - 4g_A^2 - 1\right) + q^2\left(23g_A^4 - 10g_A^2 - 1\right) + \frac{48g_A^4 m_\pi^4}{w^2}\right]L(q)\right.\\
&\quad + \left[6m_\pi^2\left(15g_A^4 - 6g_A^2 - 1\right) + q^2\left(23g_A^4 - 10g_A^2 - 1\right)\right]\ln\frac{m_\pi}{\mu}\\
&\quad \left. + 4m_\pi^2\left(4g_A^4 + g_A^2 + 1\right) + \frac{q^2}{6}\left(5g_A^4 - 26g_A^2 + 5\right)\right\},
\end{aligned} \tag{11}$$

$$V_T = -\frac{1}{q^2}V_S = -\frac{3g_A^4}{64\pi^2 f_\pi^4}L(q), \tag{12}$$

where

$$L(q) \equiv \frac{w}{q}\ln\frac{w+q}{2m_\pi}, \quad w \equiv \sqrt{4m_\pi^2 + q^2}, \quad q \equiv |\boldsymbol{q}|, \quad \boldsymbol{q} \equiv \boldsymbol{p} - \boldsymbol{p}', \tag{13}$$

with $\boldsymbol{p}, \boldsymbol{p}'$ being the external momenta for a nucleon. Below, the renormalization-scale-dependent terms ($\propto \ln\frac{m_\pi}{\mu}$) will be discarded (by putting $\mu = m_\pi$) as in Refs.[13, 28], as the qualitative status would remain the same. Besides this, the W_C of TPE given in Ref.[13] only contains the term in the first line of Eq.(11).

Now, we perform the low-energy expansion to extract contributions to the contact couplings in pionless EFT. We focus on C_0 (the superscript '($\not\pi$)' will be dropped henceforth), to which OPE contributes nothing due to the derivative πN coupling! While the TPE's contribution differs a little across different versions:

$$V_{2\pi}^{(KBW)}: \quad C_{0\tau}^{(KBW)} = -\frac{g_A^4 m_\pi^2}{8\pi^2 f_\pi^4}, \quad \Lambda_{(\not\pi,\tau)}^{(KBW)} \equiv -\frac{4\pi}{M_N C_{0\tau}^{(KBW)}} = \frac{32\pi^3 f_\pi^4}{g_A^4 M_N m_\pi^2}, \tag{14}$$

$$V_{2\pi}^{(EGM)}: \quad C_{0\tau}^{(EGM)} = -\frac{g_A^4 m_\pi^2}{12\pi^2 f_\pi^4}, \quad \Lambda_{(\not\pi,\tau)}^{(EGM)} \equiv -\frac{4\pi}{M_N C_{0\tau}^{(EGM)}} = \frac{48\pi^3 f_\pi^4}{g_A^4 M_N m_\pi^2}, \tag{15}$$

with the scale Λ thus extracted being of order 10^3 MeV (see Table 2), much larger than as the upper scale of pionless EFT that is of order $m_\pi{}^2$. According to the power counting rules of pionfull theory, the constants given in Eqs.(14,15) will be just the leading contribution to pionless

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[2]We only extracted the terms of order g_A^4 for qualitative demonstration as $g_A > 1.2$ and including the terms of lower g_A power would not alter the magnitude order of our results.

C_0 from 2N-irreducible diagrams or pion-exchange potential. Comparing with general expectation about pionless C_0, this contribution is too small. That means, the dominant contribution to the pionless coupling C_0 could not be from such irreducible diagrams. Therefore, the dominant contributions from pions to pionless C_0 could only come from the diagrams with iterations of pion-exchange potential, i.e., the 2N-reducible diagrams. The simplest case is the once-iterated OPE diagram, which has been computed long ago by the Munich group[27]. In this report, we will reanalyze it from the mapping perspective through an 'interactive' use of three-dimensional non-relativistic formulation and four-dimensional relativistic formulation.

3.2. 2N-reducible diagrams with pions: 3-dimensional non-relativistic calculation

We will basically adopt the parametrization given in Ref.[6] in our calculations. In non-relativistic formulation, the once-iterated OPE diagram reads

$$T_{1\pi}^{(it)}(p, p') = \frac{g_A^4}{16 f_\pi^4}(3 - 2\tau_1 \cdot \tau_2) \int \frac{d^3 l}{(2\pi)^3} \frac{\sigma_1 \cdot q_1 \, \sigma_2 \cdot q_1 \, \sigma_1 \cdot q_2 \, \sigma_2 \cdot q_2}{\left(q_1^2 + m_\pi^2\right)\left(q_2^2 + m_\pi^2\right)\left(E_{N;p} - \frac{l^2}{M_N} + i\epsilon\right)}, \tag{16}$$

with $q_1 = p + l$, $q_2 = p' + l$, $E_{N;p} \equiv \sqrt{p^2 + M_N^2}$.

To extract the contribution to C_0, we compute the following

$$T_{1\pi}^{(it)}(0, 0) = -\frac{g_A^4 M_N}{16 f_\pi^4}(3 - 2\tau_1 \cdot \tau_2)I_4(0), \tag{17}$$

$$I_4(0) \equiv \int \frac{d^3 l}{(2\pi)^3} \frac{l^2}{E_{\pi;l}^4}, \tag{18}$$

with $E_{\pi;l} \equiv \sqrt{l^2 + m_\pi^2}$. In standard dimensional and cutoff schemes, we have

$$I_4(0) = \begin{cases} -\dfrac{3m_\pi}{8\pi}, \ \text{(dimensional)} \\[2ex] -\dfrac{3m_\pi}{8\pi} + \dfrac{\Lambda}{2\pi^2}, \ \text{(cutoff)} \end{cases} \tag{19}$$

As will be seen in Sec.3.3, the linear divergence here is an artifact introduced by non-relativistic approximation. So, we take that

$$T_{1\pi}^{(it)}(0, 0) = \frac{3g_A^4 M_N m_\pi}{128\pi f_\pi^4}(3 - 2\tau_1 \cdot \tau_2). \tag{20}$$

This is essentially what the once-iterated OPE diagram contributes to the leading coupling C_0 in pionless theory, the contribution to $I_4(0)$ from pionless region is negligible:

$$I_4^{(\not\pi)}(0) \equiv \int_{(0, m_\pi)} \frac{d^3 l}{(2\pi)^3} \frac{l^2}{E_{\pi;l}^4} = \frac{10 - 3\pi}{16\pi^2} m_\pi = \varepsilon_4^{(\not\pi)} I_4(0), \quad \left|\varepsilon_4^{(\not\pi)}\right| = \frac{10 - 3\pi}{6\pi} \approx 3.05 \times 10^{-2} \ll 1. \tag{21}$$

Obviously, the suppression of the contribution from pionless range is due to the derivative pion-nucleon coupling.

To be more accurate, one may exclude this 3 percent in identifying the dominant contribution to C_0:

$$C_0^{(it)} + C_{0\tau}^{(it)} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \equiv T_{1\pi}^{(it)}(\mathbf{0}, \mathbf{0}) \left(1 - \varepsilon_4^{(\not\pi)}\right) \tag{22}$$

$$C_0^{(it)} = \frac{9 g_A^4 M_N m_\pi}{128 \pi f_\pi^4} \left(1 - \varepsilon_4^{(\not\pi)}\right), \quad C_{0\tau}^{(it)} = -\frac{3 g_A^4 M_N m_\pi}{64 \pi f_\pi^4} \left(1 - \varepsilon_4^{(\not\pi)}\right), \tag{23}$$

with the superscript "(it)" indicating the contribution from the once-iterated OPE diagram. Following the standard parametrization: $C_0 = \pm 4\pi M_N^{-1} \Lambda_{(\not\pi)}^{-1}$, we have

$$\Lambda_{(\not\pi)}^{(it)} = \frac{512 \pi^2 f_\pi^4}{9 g_A^4 M_N^2 m_\pi \left(1 - \varepsilon_4^{(\not\pi)}\right)}, \quad \Lambda_{(\not\pi,\tau)}^{(it)} = \frac{256 \pi^2 f_\pi^4}{3 g_A^4 M_N^2 m_\pi \left(1 - \varepsilon_4^{(\not\pi)}\right)}, \tag{24}$$

which is of the order of pion mass provided the popular choices for M_N, m_π, f_π and g_A are made. In table 1 and table 2, the 3 percent deduction is not included as it could not affect our conclusions.

3.3. 2N-reducible diagrams with pions: 4-dimensional relativistic calculation

In relativistic formulation, the once-iterated OPE diagram is contained in the following planar box diagram:

$$\begin{aligned} T^{(pb)}(\boldsymbol{p}, \boldsymbol{p}') &= \frac{g_A^4}{16 f_\pi^4} \int \frac{d^4 l}{(2\pi)^4} \frac{1}{\left(q_1^2 - m_\pi^2\right)\left(q_2^2 - m_\pi^2\right)} \bar{u}_1(\boldsymbol{p}')(-\not{q}_2)\gamma^5 \tau_1^b \frac{1}{\not{k} - M_N} \not{q}_1 \gamma^5 \tau_1^a u_1(\boldsymbol{p}) \\ &\times \bar{u}_2(-\boldsymbol{p}') \not{q}_2 \gamma^5 \tau_2^b \frac{1}{\not{k}' - M_N}(-\not{q}_1)\gamma^5 \tau_2^a u_2(-\boldsymbol{p}) \end{aligned} \tag{25}$$

with momentum flows chosen as in Ref.[6]: $q_1 = (l^0, \boldsymbol{p}-\boldsymbol{l})$, $q_2 = (l^0, \boldsymbol{p}'-\boldsymbol{l})$, $k = (E_{N;p}-l^0, \boldsymbol{l})$, $k' = (E_{N;p} + l^0, -\boldsymbol{l})$.

Again, we are interested in the situation when external momenta are zero, namely,

$$T^{(pb)}(\mathbf{0}, \mathbf{0}) = \frac{i g_A^4}{16 f_\pi^4}(3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\left[4 M_N^2 I_0 - I_2 + 16 M_N^4 I_{2+-} - 4 M_N^2(I_{2+} + I_{2-})\right], \tag{26}$$

where

$$I_0 \equiv \int \frac{d^4 l}{(2\pi)^4} \frac{1}{A_\pi^2}, \quad I_2 \equiv \int \frac{d^4 l}{(2\pi)^4} \frac{l_0^2}{A_\pi^2}, \quad I_{2+-} \equiv \int \frac{d^4 l}{(2\pi)^4} \frac{l_0^2}{A_\pi^2 A_+ A_-}, \quad I_{2\pm} \equiv \int \frac{d^4 l}{(2\pi)^4} \frac{l_0^2}{A_\pi^2 A_\pm}, \tag{27}$$

with $A_\pi \equiv l^2 - m_\pi^2 + i\epsilon$, $A_\pm \equiv l^2 \pm 2 M_N l_0 + i\epsilon$. Note that I_{2+-} is definite, the rest carry at most logarithmic divergence. In dimensional scheme, we have

$$I_0 = \frac{i}{(4\pi)^2}\left[\Gamma(\epsilon) - \ell_\pi\right], \quad I_2 = \frac{i m_\pi^2}{2(4\pi)^2}\left[\Gamma(\epsilon) + 1 - \ell_\pi\right], \tag{28}$$

$$I_{2\pm} = \frac{i}{4(4\pi)^2}\left[\Gamma(\epsilon) + 1 - \ell_N + \frac{6\varrho + (3 - 4\varrho)\ln\varrho}{2\varrho^2} + \frac{(3 - 10\varrho)\arctan\sqrt{4\varrho - 1}}{\varrho^2 \sqrt{4\varrho - 1}}\right], \tag{29}$$

$$I_{2+-} = \frac{i}{4(4\pi)^2} \frac{1}{M_N^2}\left[\frac{(1 - \varrho)\ln\varrho}{\varrho} + 2 + \frac{(2 - 6\varrho)\arctan\sqrt{4\varrho - 1}}{\varrho \sqrt{4\varrho - 1}}\right], \tag{30}$$

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with $\ell_\pi \equiv \ln \frac{m_\pi^2}{\mu^2}, \ell_N \equiv \ln \frac{M_N^2}{\mu^2}, \varrho \equiv \frac{M_N^2}{m_\pi^2}$. Then, we have

$$T^{(\mathrm{pb})}(\mathbf{0},\mathbf{0}) \;=\; -\frac{g_A^4}{128\pi^2 f_\pi^4}(3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\left[\alpha_N M_N^2 + \alpha_{N\pi} M_N m_\pi + \alpha_\pi m_\pi^2\right], \tag{31}$$

with

$$\alpha_N \;\equiv\; \Gamma(\epsilon) + 3 - \ell_N, \quad \alpha_{N\pi} \equiv \frac{(2 - 6\varrho)\arctan\sqrt{4\varrho - 1}}{\varrho\sqrt{1 - (4\varrho)^{-1}}},$$

$$\alpha_\pi \;\equiv\; -\frac{\Gamma(\epsilon) + 1 - \ell_\pi}{4} - 3 + \frac{8\varrho - 3}{2\varrho}\ln\varrho + \frac{(10\varrho - 3)\arctan\sqrt{4\varrho - 1}}{\varrho\sqrt{4\varrho - 1}}. \tag{32}$$

Obviously, the third term '$\alpha_\pi m_\pi^2$' is what one would expect for a standard TPE component of *NN* potential. The second term '$\alpha_{N\pi} M_N m_\pi$' is a definite (or nonlocal) term that comes from I_{2+-}, it will prove to be just the dominant contribution to the pionless C_0 we are after, see below. However, we are not ready yet to identify the above amplitude as contributions to the pionless coupling C_0: There is an 'offensively' large local term '$\alpha_N M_N^2$' that is completely out of control in the realm of pionfull EFT for *NN* forces. To resolve this problem, we first observe that the pionfull theory actually lives in non-relativistic regime as $\Lambda_{(\pi)}$ lies well below M_N. Then, after contour integration, M_N activates a division of loop momentum space into a low or non-relativistic region and a high or relativistic region: In the low region where non-relativistic regime is legitimately defined, expansions could be safely done with all momenta and m_π being smaller scales against M_N; While in the high region, only external momenta and m_π are smaller scales that facilitate expansions, resulting in local operators or terms of light degrees with large factors of M_N. The high region and those large local operators are automatically discarded in non-relativistic regime. Thus, '$\alpha_N M_N^2$' comes from the high region where the pionfull EFT is no longer valid and hence should be subtracted from the theory at all.

Let us illustrate with the definite integral I_{2+-} that interests us most. To enter non-relativistic regime, one first picks up the low-lying poles at $E_{N;l} - M_N \approx \frac{l^2}{2M_N}$ (nucleon) and $E_{\pi;l}$ (pion) in contour integration and then expand the resultants in terms of $1/M_N$ in the low region. For I_{2+-}, we have:

$$I_{2+-}|_{NR} \equiv \int \frac{d^3\boldsymbol{l}}{(2\pi)^3}\left(\oint \frac{dl_0}{2\pi}\frac{l_0^2}{A_\pi A_+ A_-}\right)\Bigg|_{NR} = \frac{i(4M_N I_N + I_\pi)}{64 M_N^4}, \tag{33}$$

$$I_N \equiv \int \frac{d^3\boldsymbol{l}}{(2\pi)^3}\frac{l^2}{E_{\pi;l}^4} = I_4(\mathbf{0}), \quad I_\pi \equiv \int \frac{d^3\boldsymbol{l}}{(2\pi)^3}\frac{m_\pi^4 + 4m_\pi^2 l^2 - 4M_N^2 E_{\pi;l}^2}{E_{\pi;l}^5}, \tag{34}$$

with I_N and I_π denoting the outcomes from the low-lying nucleon and pion poles, respectively. From Eqs.(18,19,30,33), we could find that, $I_4(\mathbf{0})$ actually comes from the following nonlocal term in I_{2+-}:

$$\frac{1}{64\pi^2 M_N^2} \times \frac{(2 - 6\varrho)\arctan\sqrt{4\varrho - 1}}{\varrho\sqrt{4\varrho - 1}} = \frac{1}{16M_N^3}\left\{-\frac{3m_\pi}{8\pi}\left[1 + o\left(\varrho^{-\frac{1}{2}}\right)\right]\right\}. \tag{35}$$

Here, it is transparent that the linear divergence in $I_4(\mathbf{0})$ is an artefact generated in the non-relativistic treatment of a definite (nonlocal) term in relativistic formulation, justifying our choice

for its value in Sec. 3.2. In the meantime, the following terms are automatically discarded:

$$\delta I_{2+-} = I_{2+-} - I_{2+-}|_{NR} = \frac{i}{4(4\pi)^2} \frac{1}{M_N^2} \left\{ [\Gamma(\epsilon) - \ell_N] \left(1 - \varrho^{-1} \right) + 2 + 2\varrho^{-1} + o\left(\varrho^{-\frac{3}{2}} \right) \right\}, \quad (36)$$

which are just the outcomes of the nucleon poles at $E_N \pm M_N$ integrated over the high region and some relativistic corrections. Collecting all such 'discarded' terms for $T^{(\mathrm{pb})}(\mathbf{0}, \mathbf{0})$, we have

$$\check{\Delta} T^{(\mathrm{pb})}(\mathbf{0}, \mathbf{0}) = -\frac{g_A^4}{128\pi^2 f_\pi^4} (3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \left\{ (\Gamma(\epsilon) - \ell_N) \left(M_N^2 - 4m_\pi^2 \right) + 3M_N^2 + o\left(\varrho^{-1} M_N^2 \right) \right\}, \quad (37)$$

which obviously includes the first term '$\alpha_N M_N^2$' of $T^{(\mathrm{pb})}(\mathbf{0}, \mathbf{0})$, fulfilling our earlier statement about its disposition. The above derivation makes it very clear that the disposal of the large (local) terms of nucleon mass squared is nothing but an inherent part of the operation for entering non-relativistic regime, an interesting fact lending itself to understanding the proposal for preserving the conventional power counting in relativistic baryon χPT[29].

Therefore, in non-relativistic regime, the box diagram decomposes into $2N$-reducible and $2N$-irreducible components as below:

$$T^{(\mathrm{pb})}(\mathbf{0}, \mathbf{0})\Big|_{NR} = T^{(\mathrm{pb})}(\mathbf{0}, \mathbf{0}) - \check{\Delta} T^{(\mathrm{pb})}(\mathbf{0}, \mathbf{0}) = T_{1\pi}^{(it)}(\mathbf{0}, \mathbf{0}) + V_{2\pi}^{(\mathrm{pb})}(\mathbf{0}), \quad (38)$$

with

$$V_{2\pi}^{(\mathrm{pb})}(\mathbf{0}) = \frac{g_A^4}{128\pi^2 f_\pi^4} (3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) m_\pi^2 \left\{ 4 - \frac{15}{4} [\Gamma(\epsilon) + 1 - \ell_\pi] \right\} \quad (39)$$

being the (bare) $2N$-irreducible component: part of the TPE potential[6, 27] as the crossed box diagram is not included here. Obviously, $V_{2\pi}^{(\mathrm{pb})}(\mathbf{0})$ is the outcome of the pion pole while $T_{1\pi}^{(it)}(\mathbf{0}, \mathbf{0})$ is the outcome of the low-lying nucleon pole. The divergence in $V_{2\pi}^{(\mathrm{pb})}$ is now proportional to m_π^2 and could be subtracted using the following chiral counterterms

$$\delta V_{2\pi}^{(\mathrm{pb})}(\mathbf{0}) = \frac{15 g_A^4}{512\pi^2 f_\pi^4} (3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) m_\pi^2 [\Gamma(\epsilon) + 1 - \ell_\pi] . \quad (40)$$

Now we arrive at the finite contributions to the pionless coupling C_0 from the planar box diagram that also decompose into two components

$$C_0^{(it)} + C_{0\tau}^{(it)} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \equiv T_{1\pi}^{(it)}(\mathbf{0}, \mathbf{0}) = \frac{3 g_A^4 M_N m_\pi}{128\pi f_\pi^4} (3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), \quad (41)$$

$$C_0^{(irr)} + C_{0\tau}^{(irr)} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \equiv V_{2\pi;R}^{(\mathrm{pb})}(\mathbf{0}) = \frac{g_A^4 m_\pi^2}{32\pi^2 f_\pi^4} (3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2), \quad (42)$$

with the ratio[3]

$$\frac{C_0^{(it)}}{C_0^{(irr)}} = \frac{3\pi}{4} \varrho^{\frac{1}{2}} = \frac{3\pi M_N}{4m_\pi} \approx 16.0324 \gg 1 \quad (43)$$

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[3]The prescription dependence of $V_{2\pi;R}^{(\mathrm{pb})}$ should not affect this ratio materially.

Table 1: Various contributions to $C_{0\tau}$ and $\Lambda_{(\not\pi,\tau)}$

	OPE	TPE(KBW)	TPE(EGM)	ITERATION$_\tau$
$C_{0\tau}$	0	$-\dfrac{g_A^4 m_\pi^2}{8\pi^2 f_\pi^4}$	$-\dfrac{g_A^4 m_\pi^2}{12\pi^2 f_\pi^4}$	$-\dfrac{3g_A^4 M_N m_\pi}{64\pi f_\pi^4}$
$\Lambda_{(\not\pi,\tau)}$	∞	$\dfrac{32\pi^3 f_\pi^4}{g_A^4 M_N m_\pi^2}$	$\dfrac{48\pi^3 f_\pi^4}{g_A^4 M_N m_\pi^2}$	$\dfrac{256\pi^2 f_\pi^4}{3g_A^4 M_N^2 m_\pi}$

demonstrates clearly the dominance of the $2N$-reducible component within planar box diagram. In relativistic formulation, there would be small relativistic corrections that will not alter this dominance. The crossed box diagram contains no contribution to C_0 except a $2N$-irreducible piece that belongs to TPE[6].

Here, some remarks are in order: (1) The dominant ingredient of the coupling C_0 in pionless EFT actually comes as from a definite and hence nonlocal item in the box diagram in relativistic form of pionfull theory. The non-relativistic decomposition procedure could at most bring about some sub-leading 'corrections'. The same might also happen to higher pionless couplings. (2) Through 'interactive' use of non-relativistic (lower) and relativistic (higher) theories, we also identified the rationale for discarding 'offensively' large terms present in the relativistic formulation by exploiting their virtues. Recently, the virtue that relativistic formulation embodies less UV divergences has also been exploited in Ref[30], resulting in a modified Weinberg approach for nuclear forces where former pathologies could be removed or diminished. (3) Therefore, the following strategy is adopted in our derivations: a) In relativistic form, we separate out and discard the high region contributions to stay in non-relativistic regime[4], the rest will be chiral divergences ($2N$-irreducible diagrams) that could be subtracted using counterterms of chiral effective theory; b) In non-relativistic form, the new (power) divergences in the $2N$-reducible diagrams are artefact of non-relativistic truncation and hence treated with dimensional regularization, the $2N$-irreducible ones fulfill standard chiral perturbation subtractions.

The various contributions to the pionless $C_{0\tau}$ are summarized in table 1 and table 2. In table 2, we also listed the scale extracted for the isospin-independent coupling $C_0^{(it)}$ in the last column.

4. Region division, enhancement and mapping

4.1. General reasoning

In relativistic formulation of any EFT, loop momentum scale extends to infinity. However, the vast region above the upper scale of EFT, $[\Lambda_{(EFT)}, \infty)$, is actually superfluous. For theories with

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[4]Discarding such contributions and this high region entirely is in fact applying a 'projection' operation to the relativistic formulation to single out non-relativistic components for further treatment, not the standard renormalization that leaves quite some ambiguities.

Table 2: Values of $\Lambda_{(\not\pi,\tau)}$ (and $\Lambda_{(\not\pi)}$) in MeV with $(f_\pi, m_\pi, M_N) = (92.4, 138, 939)$ MeV.

g_A	TPE(KBW)	TPE(EGM)	ITERATION$_\tau$	ITERATION
1.26	1604.65 $(\sim 11.63 m_\pi)$	2406.98 $(\sim 17.44 m_\pi)$	200.18 $(\sim 1.45 m_\pi)$	133.45 $(\sim 0.97 m_\pi)$
1.29	1460.51 $(\sim 10.58 m_\pi)$	2190.77 $(\sim 15.88 m_\pi)$	182.20 $(\sim 1.32 m_\pi)$	121.46 $(\sim 0.88 m_\pi)$
1.32	1332.20 $(\sim 9.65 m_\pi)$	1998.29 $(\sim 14.48 m_\pi)$	166.19 $(\sim 1.20 m_\pi)$	110.79 $(\sim 0.80 m_\pi)$

light mass scales, the vast superfluous region is of no harm. Things become complicated when an EFT actually lives in non-relativistic regime: Offensively large terms have to be separated out and subtracted to stay in non-relativistic regime and intricacies arise due to the infrared enhancement in non-relativistic regime. In the pionfull theory for nuclear forces, the pions mass facilitates a further division of the low region '$U_{(\pi)}$' into pionless region '$U_{(\not\pi)}$' and its complement '$\tilde{U}_{(\pi)}$':

$$U_{(\pi)} = U_{(\not\pi)} \cup \tilde{U}_{(\pi)}, \quad U_{(\not\pi)} \equiv [0, \Lambda_{(\not\pi)}), \quad \tilde{U}_{(\pi)} \equiv [\Lambda_{(\not\pi)}, \Lambda_{(\pi)}). \tag{44}$$

Intricacies actually lie in $\tilde{U}_{(\pi)}$, where low-lying nucleon poles dominate the contributions to pion-less couplings and give rise to infrared enhancement at least in planar box diagram[5]. Of course, it remains to see how higher diagrams behave in this region, especially how the low-lying nucleon poles in these diagrams contribute to pionless couplings!

Technically, the dominance of iterated OPE over TPE (and of course, OPE) is due to the dominance of $4M_N I_N$ over I_π, which in turn comes from the fact that the low-lying nucleon poles tend to pinch. We also need that the contribution from the pionless region to I_N is negligible, which is guaranteed by the derivative coupling between pion and nucleons. Thus, for the anomalous dominance of reducible diagrams to happen, we need: (1) non-relativistic regime to make the low-lying nucleon poles tend to pinch; (2) derivative coupling between pion and nucleons to suppress the contributions from pionless region so that the pionfull region $\tilde{U}_{(\pi)}$ holds the bulk contributions, (3) clear separation of mass scales to make the enhancement materialize, i.e., $\sqrt{\varrho} \gg 1$. Of course, there is a gross prerequisite here: the large relativistic components must be entirely excluded or discarded in the first place. Otherwise, the whole theory will be overwhelmed by the high region, which is totally unacceptable. Unless profound changes are made to substantially invalidate the above three features or conditions, the dominance of iterated

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[5]Literally, as high and low regions are separated by nucleon mass M_N, an extra region $\delta U_{low} = [\Lambda_{(\pi)}, M_N)$ is implictly included in the loop integration in non-relativistic decomposition. We are not clear yet the roles played by this extra region.

diagrams is doomed to happen, right within the region $\tilde{U}_{(\pi)}$[6]. So, in this perspective, the real issue with the pionfull theory for nuclear forces stems from the low region of $\tilde{U}_{(\pi)}$, not the high region that must be separated out and discarded. Hence, one either works entirely in nonperturbative regime to accommodate the dominance of iterated diagrams or profoundly alters the organization of the theory right in the region $\tilde{U}_{(\pi)}$ to remove or reduce the infrared enhancement.

In addition, our primary analysis using once-iterated OPE diagram seems to support the conventional power counting rules for pionless EFT given in Sec. 2.2. Of course, we could not make conclusive statement yet before contributions from higher diagrams are comprehensively studied. Below, we wish to see what could happen to the pionfull-pionless mapping in BKV prescription.

4.2. Mapping in BKV prescription

In BKV prescription, the higher modes are separated out from OPE using the following means [26]:

$$V_{1\pi}^{(\text{BKV})}(\boldsymbol{q}) = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\boldsymbol{\sigma}_1 \cdot \boldsymbol{q}\, \boldsymbol{\sigma}_2 \cdot \boldsymbol{q} \left(\frac{1}{q^2 + m_\pi^2} - \frac{1}{q^2 + \lambda_{\text{BKV}}^2} \right) + \frac{\lambda_{\text{BKV}}^2}{q^2 + \lambda_{\text{BKV}}^2} \right], \tag{45}$$

with λ_{BKV} (set at 750MeV) being the separation scale. Obviously, this 'OPE' contributes to the leading coupling:

$$C_{0\tau}^{(\text{BKV})}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = V_{1\pi}^{(\text{BKV})}(\boldsymbol{0}) = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \tag{46}$$

with '$\frac{4\pi}{M_N C_{0\tau}}$' being close to $2m_\pi$, so it will be taken as the leading contribution to $C_{0\tau}$ from pion-exchange potential as higher order pion-exchange (TPE, etc.) should also be sub-leading in BKV prescription.

In the meantime, the iteration of this 'OPE' gives ($\theta \equiv \frac{\lambda_{\text{BKV}}}{m_\pi}$):

$$\begin{aligned}
T_{1\pi}^{(it;\text{BKV})}(\boldsymbol{0},\boldsymbol{0}) &= -\frac{g_A^4 M_N m_\pi}{16f_\pi^4}(3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\left[I_{4;(\text{BKV})}(\boldsymbol{0}) + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 I_{4\sigma;(\text{BKV})}(\boldsymbol{0})\right] \\
&= -\frac{g_A^4 M_N m_\pi}{16f_\pi^4}(3 - 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\left\{\frac{2\theta^2 - \theta + 1}{8\pi(1 + \theta)} + \frac{\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \theta^2}{6\pi(1 + \theta)}\right\},
\end{aligned} \tag{47}$$

from which we could find (using $\lambda_{\text{BKV}} = 750$MeV)

$$\left|\frac{C_{0\tau}^{(it;\text{BKV})}}{C_{0\tau}^{(\text{BKV})}}\right| = \frac{g_A^2 M_N m_\pi}{16\pi f_\pi^2}\frac{2\theta^2 - \theta + 1}{(1 + \theta)} \approx 2.5639 g_A^2, \tag{48}$$

which moves from 4.07 to 4.47 as g_A varies from 1.26 to 1.32. In this case, $I_{4;(\text{BKV})}$ still mainly comes from the pionfull region as $I_{4;(\text{BKV})}(< m_\pi)/I_{4;(\text{BKV})} \approx 15.6\%$. Excluding the pionless region, the above ratio becomes $\approx 2.1632 g_A^2$, which moves from 3.43 to 3.77 as g_A varies from 1.26 to 1.32. In any case, the dominance of the iterated OPE or $2N$-reducible diagram still happens,

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[6]This is also reflected by the fact that the scale $\sqrt{M_N m_\pi}$ from infrared enhancement is close to half of $\Lambda_{(\pi)}$: $\sqrt{M_N m_\pi} \sim 2.61 m_\pi \sim \frac{\Lambda_{(\pi)}}{2}$.

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leading again to anomalous mapping between pionfull and pionless theories. This is because that derivative πN coupling in BKV 'OPE' still controls the low or pionfull region $U_{(\pi)}$, as the modification here is mainly introduced to tame the UV behavior of OPE in triplet channels. That means, the three conditions for dominance of iterated diagrams still hold.

Here, we note in passing that the scaling of the dominant contribution to pionless C_0 in BKV is approximately $C_{0\tau}^{(it;\text{BKV})} \sim \frac{4\pi}{M_N \Lambda_{(\not\pi;\text{BKV})}}$ with $\Lambda_{(\not\pi;\text{BKV})} \approx \frac{1}{2} m_\pi$, quite a distance from the KSW scaling $C_0 \sim \frac{4\pi}{M_N Q}$ with $Q \sim 35\text{MeV}$, thus the KSW scaling for pionless EFT is not quite realized yet in the once-iterated 'OPE' of BKV prescription. Of course, further studies of higher diagrams are needed for a conclusive judgement.

4.3. Emergence of large scattering lengths

Our foregoing derivation clearly demonstrated that the pionless coupling C_0 essentially comes from a definite item that assumes contributions from the region $[\Lambda_{(\not\pi)}, \Lambda_{(\pi)})$ in the (underlying) pionfull theory. Therefore, loop integrations in pionless EFT only make sense over the pionless region $[0, \Lambda_{(\not\pi)})$ as contributions from $[\Lambda_{(\not\pi)}, \Lambda_{(\pi)})$ have been collected into the pionless couplings. In this perspective, the pionless integrals would become definite items in the (underlying) pionfull theory that collect contributions from the pionless region $[0, \Lambda_{(\not\pi)})$ only. For example, in $T_0 = \frac{C_0}{1 - C_0 I_{0;(\not\pi)}}$ generated by the pionless C_0, the pionless integral

$$I_{0;(\not\pi)} \equiv \int \frac{d^3 l}{(2\pi)^3} \frac{1}{E - l^2/M_N + i\epsilon} = -J_0 - i\frac{M_N p}{4\pi} \quad \left(p \equiv \sqrt{M_N E}\right)$$

should collect contributions from the region $[0, \Lambda_{(\not\pi)})$ in a well-defined manner in pionfull theory, hence $J_0 \sim \frac{M_N}{4\pi} \Lambda_{(\not\pi)}$. Then large scattering lengths in S waves would 'naturally' emerge provided $C_0 \sim -\frac{4\pi}{M_N} \Lambda_{(\not\pi)}^{-1}$:

$$\frac{1}{a} = \Re\left[-\frac{4\pi}{M_N T_0}\right]_{p=0} = -\frac{4\pi}{M_N}\left(\frac{1}{C_0} + J_0\right) = \pm o\left(\epsilon^\sigma \Lambda_{(\not\pi)}\right), \left(\sigma \geq 1, \epsilon \sim 4^{-1}\right)$$

which is true even after higher couplings are included[21–24]. That is, the large scattering lengths arise from the 'cancelation' between C_0^{-1} and J_0 that are effective measures of certain intrinsic properties of the pionfull theory. Hence, it is intriguing to extract the pionless parameters like J_0 from pionfull theory as we did for pionless couplings, i.e., to calculate J_0 from mapping perspective. We will pursue such studies in future. The above perspective for couplings and loop integrations in an EFT with an upper scale should be generally true in EFT descriptions of many systems and useful for renormalization in various EFT contexts.

5. Prospective studies and summary

So far we have just performed some primary analysis about the pionfull effective theory in mapping perspective. Obviously, there are a lot more works to be done in the future. (1) Generically, pionless couplings without derivatives take the following form: $C_0 + C_{0\tau}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + C_{0\sigma}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + C_{0\sigma\tau}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ or $\tilde{C}_0 + \tilde{C}_{0\sigma}\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$ after using Fierz transformation. Firm conclusions about the values of these constants could only be drawn after higher loop diagrams are comprehensively studied. The same is true for the rest contact couplings with derivatives. Moreover, it is also interesting to study the extraction of the parameters like J_0 and all that

appear in pionless integrals from pion-exchange diagrams. From such studies, we could learn more about the pionfull effective theory for nuclear forces and pin down the scenario for pionless theory as a byproduct. (2) It remains to see if the strategy for renormalization suggested here that spares us from twisting the chiral power counting rules could work out at higher orders and/or in nonperturbative regime. In other words, it is interesting to see if the observation that power divergences in non-relativistic formulation are merely artefact from non-relativistic treatment could be developed into a set of working rules in future for doing calculations in non-relativistic regime in massive systems. In particular, it would be interesting to see how this observation and the mapping perspective could be applied to 3N or multi-body nuclear forces. (3) Enhancement in pion-exchange diagrams has recently been exploited to study $N\bar{N}$ systems near production threshold in Refs.[31] with an effective field theory similar to that for NN system being developed and employed there. It will be interesting to explore the detailed mechanism in such effective theories and related systems to gain further insights into pionfull effective theories.

As far as OPE is concerned, the infrared enhancement inherent in the iterated OPE diagrams is the main driving force for working in nonperturbative regimes. It is also the driving force for developing approaches that incorporate this enhancement to certain extent in various manners. For example, infrared enhancement is at least partially incorporated into the low-momentum effective potentials constructed by integrating out modes above the scale $\Lambda \approx 2.1\text{fm}^{-1}$[32], as this benchmark scale sits right in the middle of the pionfull range $\tilde{U}_{(\pi)}$: $\frac{m_\pi + \Lambda_{(\pi)}}{2} \approx 3m_\pi \approx 2.1\text{fm}^{-1}$, it is also close to the enhancement scale $\sqrt{M_N m_\pi} \approx 1.8\text{fm}^{-1}$. In this connection, it is interesting to see how the enhancement is alleviated or affected through introducing other degrees like dibaryons[33]. We will consider this else where. Furthermore, if one wish to remove the infrared enhanced items from iterated OPE, then: (1) The contact couplings in pionfull theory must be promoted up to absorb such enhanced items; (2) The iteration diagrams must be accordingly modified to avoid double counting. Endeavors in this direction might lead us to a 'perturbative' approach that is similar to KSW's somehow. In short, it is worthwhile to take an alternative look at the important issue of pion-exchange potential in nonperturbative regime. For continuing efforts being invested in the two popular choices mentioned in the beginning, see, e.g.,[34, 35]. Aside from what we discussed above, there might be other sources of intricacy in pionfull effective theory, to name one, the nature of sigma meson [36] and its couplings to pions and nucleons may also play some unknown roles in the pionfull effective theory for nuclear forces. In a sense, the intricacy met so far may imply that there are still sophisticated structures of hadronic dynamics to be uncovered besides the celebrated chiral effective field theory based on chiral symmetry realized nonlinearly.

In summary, we performed a primary analysis of the mapping between pionfull and pionless effective field theories for nuclear forces through interactive use of non-relativistic and relativistic formulations. The dominant contribution to the pionless coupling C_0 is provided by the $2N$-reducible component of the planar box diagram which becomes a definite item in relativistic formulation, not from the $2N$-irreducible component. This anomalous mapping is due to the enhancement generated by the low-lying nucleons poles and also happens in the BKV modification of OPE. As a byproduct, a simple strategy for renormalizing the pionfull theory surfaced from our combined analysis in relativistic and non-relativistic formulations. Prospective studies are addressed and related issues discussed.

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